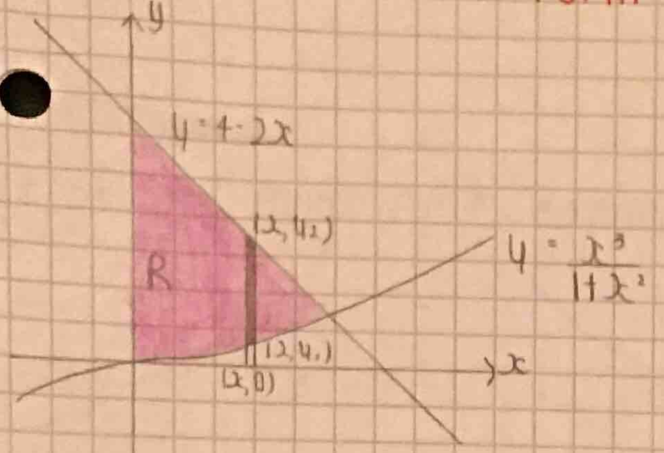
2002: PART A Form B

1)



a) $4 - 2x = \frac{x^3}{1+x^2}$

$x = 1.487664$

$\text{Area } R = \int_0^{1.487664} \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$

$= 3.215$

b) $R(x) = y_2 - 0 = 4 - 2x$

$r(x) = y_1 - 0 = \frac{x^3}{1+x^2}$

$\text{Volume} = \pi \int_0^{1.487664} \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right) dx$

$= 31.885$

c) square

$4 - 2x - \left(\frac{x^3}{1+x^2}\right)$

$\text{Volume} = \int_0^{1.487664} \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$

$= 8.997$

2) $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$

a) $P'(9) = 1 - 3e^{-0.2\sqrt{9}} = -0.646435 < 0$

$\therefore$ amount is not increasing at this time

b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$3e^{-0.2\sqrt{t}} = 1$

$e^{-0.2\sqrt{t}} = 1/3$

$-0.2\sqrt{t} = \ln(1/3)$

$-0.2\sqrt{t} = -\ln 3$

$0.2\sqrt{t} = \ln 3$

$t = 5\ln 3$

$t = (5\ln 3)^2 = 30.174$

$P'(t)$: $-$ | $+$

0   30.174

\   —   /

Minimum at $t = (5\ln 3)^2$ since $P'(t)$ changes sign from $-$ve to $+$ve

c) $P(30.174) = 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}})\,dt$

$= 30.104$

Since $30.104 < 40$, lake is considered safe

d) Equation of tangent: $(0, 50)$, $P'(0) = 1 - 3e^0 = -2$

$y - 50 = -2(t - 0)$

$y = -2t + 50$

$-2t + 50 = 40$

$t = 5$ days

3) $v(t) = e^{2\sin t} - 1$, $0 \le t \le 16$

b) Particle is moving to the left when $v(t) < 0$

$e^{2\sin t} - 1 < 0$

$e^{2\sin t} < 1$

$2\sin t < \ln 1$

$\sin t < 0$

$+$ $-$ $+$ $-$ $+$ $-$

0 π 2π 3π 4π 5π 16

$(\pi, 2\pi), (3\pi, 4\pi), (5\pi, 16)$

c) Total Distance $= \int_0^{\pi} v(t)\,dt - \int_{\pi}^{4} v(t)\,dt$

$= 10.542$

d) $\int_0^{T} v(t)\,dt > 0$ for all $T > 0$

002: PART B Form B

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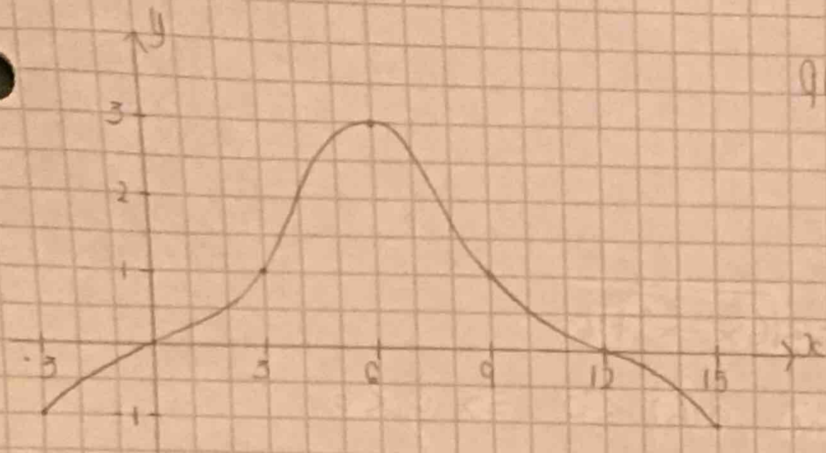


Graph of f

$g(x) = 5 + \int_6^x f(t)\,dt$

a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

b) $f = g'(x)$

g is decreasing on $[-3,0]$ and $[12,15]$ since $g'(x) < 0$

c) concave down on $(6,15)$ since $g'(x) = f$ is decreasing on this interval

d) $\int_{-3}^{15} f(t)\,dt = \frac{15+3}{2(6)}\left[f(-3) + 2\left(f(0)+f(3)+f(6)+f(9)+f(12)\right) + f(15)\right]$

$= \frac{3}{2}(-1 + 2(0+1+3+1+0) + (-1))$

$= \frac{3}{2}(-1 + 10 - 1)$

$= \frac{3}{2} \cdot 8$

$= 12$

5) $\frac{dy}{dx} = \frac{3-x}{y}$

a) $\frac{dy}{dx} = 0$ when $y = -2$

$\frac{3-x}{-2} = 0$

$3 - x = 0$

$x = 3$

$\frac{d^2y}{dx^2} = \frac{y(-1) - (3-x)y'}{y^2} = \frac{-y - (3-x)y}{y^2}$

at $(3,-2)$ $\frac{d^2y}{dx^2} = \frac{2}{4} = \frac{1}{2} > 0$ $\therefore$ f has a local minimum at this poin

or $\frac{dy}{dx}$: $-$ $+$

1 3 5

b) $\int y\,dy = \int (3 - x)\,dx$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C$

$y^2 = 6x - x^2 + C$

$y(6) = -4$

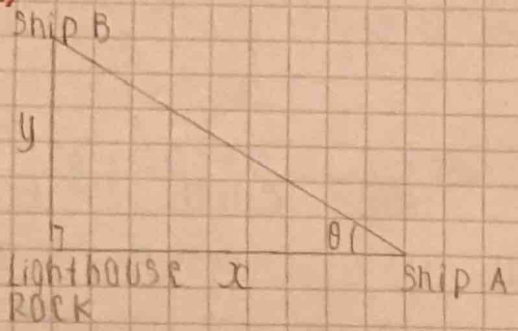
$(-4)^2 = 6(6) - 6^2 + C$

$C = 16$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

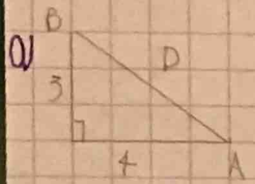
6)



$\frac{dx}{dt} = -15$ km/hr

$\frac{dy}{dt} = 10$ km/hr

a)



$D^2 = 4^2 + 3^2$

$D^2 = 25$

$D = 5$

5 km

b) We want $\frac{dD}{dt}$ when $x = 4,\ y = 3$

$D^2 = x^2 + y^2$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$2(5)\frac{dD}{dt} = 2(4)(-15) + 2(3)(10)$

$10\frac{dD}{dt} = -60$

$\frac{dD}{dt} = -6$ km/hr

c) We want $\frac{d\theta}{dt}$ when $x = 4,\ y = 3$

$\tan\theta = \frac{y}{x}$

$\sec\theta = \frac{5}{4}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$

$\left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{16} \Rightarrow \frac{25}{16}\frac{d\theta}{dt} = \frac{85}{16} \Rightarrow \frac{d\theta}{dt} = \frac{85}{25} = \frac{17}{5}$ radians/hr